Line integrals

Questions

Question 1. Determine if each of the following vector fields is conservative. If they are, find a potential function.

(a) $\mathbf{F} = \langle 3x^2 + y^2, -2xy \rangle$
(b) $\mathbf{F} = \langle 3x^2 - y^2, -2xy \rangle$
(c) $\mathbf{F} = \langle 3x^2 + y^2, 2xy \rangle$
(d) $\mathbf{F} = \langle 3x^2 - y^2, 2xy \rangle$

Question 2. Evaluate the line integral $\int_C (\sin x \, dx + \cos y \, dy)$, where C consists of the top part of the circle $x^2 + y^2 = 1$ from $(1, 0)$ to $(-1, 0)$, followed by the line segment from $(-1, 0)$ to $(2, -\pi)$.

Question 3. Let C be the portion of the curve $x = y^2/2$ in the range $-2 \le y \le 1$. Evaluate

$$\int_C (y + 2xy) \, ds.$$